

Source terms from two-phase flow in long pipelines following an accidental breach

D M Webber¹, T K Fannelop², and H W M Witlox³

¹ Integral Science and Software Ltd, Warrington, UK

² E & E Flow Analysis, Aasgaardsstrand, Norway

³ Det Norske Veritas, London, UK

Abstract

Pressure-liquefied gases are routinely transported along pipelines many kilometres in length. The possibility of an accidental breach of such a pipeline forms a potentially severe hazard, which may lead to a large dispersion source term or more directly to a fire or explosion.

Models of flows in long pipelines exist which are complicated (relying on a full CFD solution of time-dependent Reynolds equations), or applicable only to gases, or applicable (explicitly or implicitly) only to simple thermodynamics and relatively small pressure drops.

One of the substances of interest is ethylene, with a critical temperature around 10 Celsius, and which may be pumped along pipes at pressures approaching 100 Bar. We have therefore derived a mathematical two-phase flow model, which admits an appropriate thermodynamic description and is formulated in such a way that no thermodynamic singularities appear at the critical point. Furthermore it is constructed without recourse to any simplifying assumptions which might become less accurate at large pressure drops, and covers both choked and unchoked flow conditions. It therefore applies to any fluid which boils significantly below ambient temperature. Nevertheless it is a simple, explicit, and transparent integral model involving clear physical assumptions. It leads to predictions of the evolution of the pressure and temperature at the outlet and at the upstream end of the pipe and of the mass flow rate out of the breach, which depend on no free parameters. Computationally it requires only a knowledge of a few thermodynamic properties of the substance involved, and the ability to do explicit definite integrals - for example by Simpson's rule.

We show that the model fits experimental data on propane releases from a 100m pipe breached suddenly at one end (the Shell "Isle of Grain" trials) at least as well as more complicated models. In summary this paper presents a new model including all relevant physics, but which is simple and requires little CPU time. It can be used routinely by engineers for practical situations.

1 Introduction

We report here on the development of a mathematical model of two-phase fluid flow in a pipeline following an accidental breach. Two-phase flow will be important in any breach of containment of gases which are stored or transported as pressurised liquids. A specific objective is a model which will comprehend the behaviour of ethylene, which may be pumped at pressures approaching 100 Bar and which has a critical temperature around 10 Celsius. It is therefore important that our model should not break down at large pressure differences or in the vicinity of the critical point or for very long (many km) pipelines.

It is also important that the model should comprehend the behaviour of propane at much more modest pressures, with a critical point further removed from the thermodynamic region of interest, and in shorter (100m) pipes as the Shell Isle of Grain experiments on propane (Cowley and Tam 1988) constitute a very useful data set for model validation. These data have recently been summarised by Richardson and Saville (1996) who also compare their model with the data and set a standard for the kind of agreement which is possible. However, from their qualitative description, their model is apparently rather complex and they place great weight on an accurate understanding of the thermodynamics of multicomponent hydrocarbons. We shall derive a simpler model, and in our concluding sections we shall discuss it with reference to the Isle of Grain data, and in comparison with the single-phase gas flow model of Fannelop and Ryhming (1982) and the two-phase flow models of Leung and co-workers (eg Leung 1986) and of Richardson and Saville (1996).

2 The Model

Our model postulates that following a breach in the pipeline there will in general be four successive flow regimes.

Regime i - rapid depressurisation. Immediately following the breach, the pressure in the pipe will decrease from the operating pressure very rapidly. The rate of drop will be governed by the speed of sound in the liquid (or supercritical fluid) in the pipe and we assume that this process ends with the fluid as saturated liquid at some temperature T_0 and pressure $p_0 = p_{\text{sat}}(T_0)$. Thermodynamically this process may be complicated by shock waves in the pipe, and there is some uncertainty in the degree of cooling expected. Dynamically it may be slightly affected by the non-rigidity of the pipe itself (elasticity of the pipe wall). However, we expect any expansion, and hence ejection of fluid from the breach, to be governed primarily by the compressibility of the liquid, which is small, especially on the scale of later expansion due to vaporisation. Our estimates of this regime therefore result in the conclusion that this process happens very quickly compared with what follows, and that the loss of fluid is small, also compared with what follows.

We shall also assume that during this regime the fluid comes effectively to rest (if any pumped inflow is cut off by valve closure) or to a flow rate governed by the pump (if it is not). In fact this process is likely to be rather violent and one must question the survivability of any pumps and valve mechanisms, but that depends on their design and is outside the scope of this work.

Estimating the temperature T_0 achieved during this regime may involve some uncertainty as it is unlikely to be as simple as isentropic, isenthalpic, or isothermal expansion. However, for the purposes of this paper, we shall simply note that in the Isle of Grain trials the propane pressure appears to have dropped very rapidly with no significant temperature decrease - but we also observe for future reference that the pressure drop from about 11bar to the saturated vapour pressure of around 8bar is relatively modest compared with what may be expected in some situations of interest.

Regime ii - flash front propagation Following regime i, we have a pipe containing saturated liquid at rest or being pumped at some given flow rate. It will start to vaporise and depressurise further, extracting heat from the fluid itself and the surrounding pipe wall. We expect vaporisation to start at the breach where the fluid sees the ambient pressure environment and, considering for the moment a pipe closed at its upstream end, our model for this regime is one of two fluid zones as shown in figure 1.

The upstream zone (Zone 1) is saturated liquid at rest at constant temperature T_0 and pressure $p_0 = p_{\text{sat}}(T_0)$. Fluid is flowing from the interface (b) to the breach (e) and as it moves downstream, the gas fraction increases and the pressure decreases to some value p_e at (e). The flow rate through Zone 2 is coupled with the pressure drop and the rate of expansion due to vaporisation between (b) and (e). As the mass of fluid in the pipe decreases, the Zone interface (b) moves upstream, and forming a flash front which erodes the liquid zone 1.

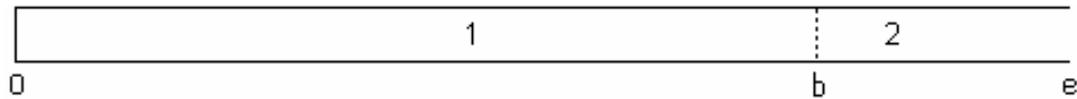


Fig. 1 Flow in regime ii in a pipe closed at the upstream end. Zone 1 is saturated liquid at rest. Zone 2 is two-phase fluid flowing from left to right and out of the breach (e). The gas fraction in Zone 2 increases from zero at the zone interface (b) as one goes downstream to the breach (e).

Initially at least, the flow is expected to be choked at (e) and the exit pressure p_e will be greater than the ambient pressure p_a . At the onset of regime ii the saturated liquid is assumed to fill the pipe. That is to say that the flash front (b) initially coincides with the exit (e) and the exit pressure p_e starts at p_0 . In order to achieve this consistently, the initial flow rate out of the exit must be that which corresponds with a choke pressure equal to p_0 , and, as we shall see, this criterion alone gives a remarkably accurate prediction of the initial rate of mass loss.

Ultimately the flash front (b) will encounter the closed upstream end of the pipe, and this signals the end of regime ii and the onset of regime iii.

If, instead of a closed upstream end, we have a constant inflow, then the flash front may ultimately be arrested at some point in the pipe when the outflow at the breach balances the inflow. In this case a steady flow will be set up and regime iii will not be relevant.

Regime iii - continuing two phase flow. After the flash front encounters the closed upstream end of the pipe, then the two-phase zone, Zone 2, will fill the pipe. Only at this point will the upstream pressure start to drop. It will continue to drop until both the upstream and exit pressures reach the ambient value and there is no thermodynamically induced pressure gradient to drive flow out of the pipe.

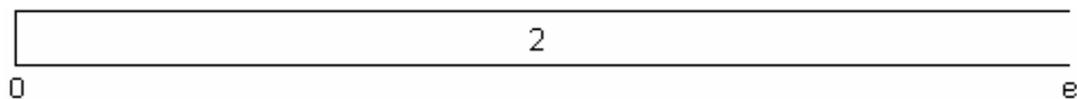


Fig. 2 Flow in regime iii in a pipe closed at the upstream end. The flash front has reached the upstream, closed end of the pipe and the two-phase zone, Zone 2, now fills the entire pipe. There is still a significant gradient of pressure and liquid fraction along the pipe, and vaporisation is still driving fluid out of the breach.

Regime iv - unpressurised flow. At the end of regime iii, almost all the mass of fluid will have been forced out of the pipe. The liquid component of any small amount remaining will be expected to settle in the pipe and any further flow will be governed by slow evaporation from the now cold pipe and gravity flow. The mass release rate will be very small compared with that in regimes ii and iii and we shall neglect it here.

Thus we see, from the point of view of hazard analysis, that regimes ii and iii are the most important ones to consider in detail and this is where our detailed modelling below will apply.

Before going on to define the model quantitatively, one or two comments are in order. Firstly we have also considered the possibility that there may be a third zone in the pipe, downstream of Zone 2, in which all the liquid has vaporised and we have pure gas flow. However, the heat available from the liquid itself is usually insufficient to achieve this in fluids of interest and our estimates of possible heat transfer rates from the pipe wall also lead us to believe that the fluid will remain in a two-phase state for the entire time under

consideration. Secondly, the assumption of the above flow regimes is we believe very plausible, and is what renders the calculations below so tractable. Further assumptions below are very much within this framework. The framework itself can be tested in principle, for example by applying a large Computational Fluid Dynamic code to the problem, which makes no such assumptions, and examining the results to see to what extent the regimes ii and iii can be identified. However, we believe there is some fairly direct evidence for these regimes from the Isle of Grain experiments, as we shall discuss below.

Finally we should emphasise that we are explicitly considering approximately horizontal pipelines. However in regimes ii and iii where the mass release rates are greatest, the mass is effectively pushed out by the thermodynamic forces vaporising the liquid. These can be expected to dominate the gravitational forces as long as the gravity head is not too large. Gravity effects may be very important in regime iv, but the mass released in that regime will be small for the two-phase flows we are considering. Therefore we shall neglect gravity. (The importance of gravity can be estimated post-hoc simply by comparing the predicted pressure differences with the gravity head.)

3 Homogeneous Equilibrium Thermodynamics

Experimental evidence

Before looking at the fluid dynamic aspects of the problem in more detail, we know we are going to need to make assumptions about the thermodynamic nature of the two-phase flow. A popular, and attractively simple, assumption is that the phases remain in thermodynamic equilibrium. But this assumption is so crucial to the flow dynamics that it would be reassuring to have some experimental evidence that this may be the case. Happily there is some very good evidence. The Isle of Grain propane experiments presented measurements of the temperature and pressure at the ends of the pipe and how they varied together in time. It is a simple matter to plot the measurements of pressure against temperature and superimpose these on the known vapour pressure curve of propane (Reid, Prausnitz and Poling 1987.)

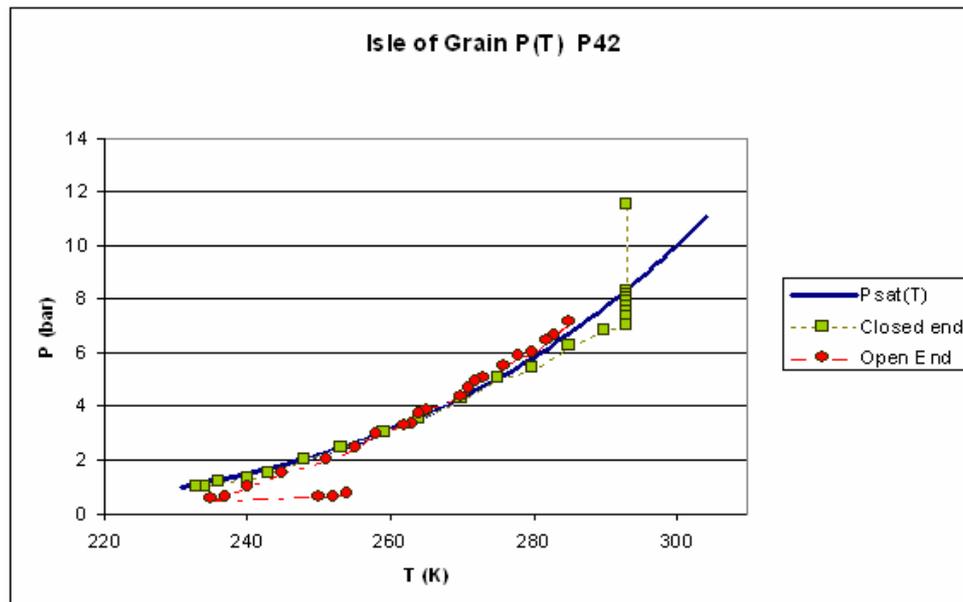


Fig. 3 Pressure and temperature measured at the closed and open ends of the pipe in Isle of Grain Trial P42 superimposed on the saturated vapour pressure curve $p=p_{\text{sat}}(T)$ of propane.

We show the results for trial P42 in Figure 3, in which the line represents the vapour pressure curve of propane, and the square and diamond points represent the experimental measurements at the upstream

(closed) and downstream (open) ends respectively. The time evolution in the experiments, from top right to bottom left, is indicated by the broken lines .

The results of this plot are very revealing. First the $p(T)$ relationship at both ends is spectacularly close to the vapour pressure curve and we need have no further doubts about using the homogeneous equilibrium model, at least for the propane pipeline used in these experiments.

There are two areas where the experimental values depart from the curve. At the top right, the first point at the closed end represents the initial pressure which was well above the saturated vapour pressure at the (ambient) initial temperature. The immediate and apparently isothermal (in this case) pressure drop to the vapour pressure curve is exactly what we have described in regime i above. From then on the results adhere to the vapour pressure curve until at the open end there is ultimately an increase in temperature at constant ambient pressure. Examination of the mass inventory at this point shows that essentially all of the mass has been expelled from the pipe by this time, and that we are seeing a warming after the experiment is effectively over and the liquid has vaporised - regime iv above.

We regard this as very strong support for the homogeneous equilibrium model (in the two-phase flow Zone) which we shall adopt below. It is also entirely consistent with the flow regimes assumed above.

Equations of state

Adopting the homogeneous equilibrium model for all two-phase flow zones of regimes ii and iii allows us to relate temperature and pressure everywhere by

$$T = T_{sat}(p) \quad or \quad p = p_{sat}(T) \quad (3.1)$$

The specific volume v and specific enthalpy h can both be expressed in terms of the local liquid mass fraction η as

$$v = \eta v_L + (1 - \eta)v_v \quad (3.2)$$

$$h = \eta h_L + (1 - \eta)h_v \quad (3.3)$$

where subscripts L and v refer to the liquid and vapour components. In fact we shall not need the mass fraction explicitly in our calculations, and eliminating it gives

$$h = h_L + (v - v_L)\phi \quad (3.4)$$

where

$$\phi(T) \equiv \frac{h_v - h_L}{v_v - v_L} = T \frac{dp}{dT} \quad (3.5)$$

The last equality (the Clapeyron Equation) is an exact, general thermodynamic property, and it means that the thermodynamic properties needed for this model will almost entirely be restricted to the vapour pressure curve $p=p_{sat}(T)$, the liquid specific volume at saturation $v_L(T)$, the liquid specific enthalpy $h_L(T)$ at saturation. These are assumed known, and in particular ϕ is a thermodynamic property (with the dimensions of pressure and which is defined along the vapour pressure curve) which will appear throughout our model as it defines the homogeneous equilibrium model's two-phase equation of state for $h(T,v)$.

4 One-dimensional flow

The model

Our starting point is a one-dimensional flow model, which may be described by the mass, momentum, and energy equations. Denoting t =time and x =downstream distance, we have

$$\frac{\partial}{\partial t} \left(\frac{1}{v} \right) + \frac{\partial G}{\partial x} = 0 \quad (4.1)$$

$$\frac{\partial G}{\partial t} + \frac{\partial(G^2 v + p)}{\partial x} = -2f \frac{G |G| v}{D} \quad (4.2)$$

$$\frac{\partial([h + pv + G^2 v^2 / 2] / v)}{\partial t} + \frac{\partial([h + G^2 v^2 / 2]G)}{\partial x} = \frac{4q}{D} \quad (4.3)$$

in which we have focused on the mass flux density G [dimensions $ML^{-2}t^{-1}$], the fluid specific volume v [$L^3 M^{-1}$], pressure p [$ML^{-1}t^{-2}$] and the fluid specific enthalpy h [$EM^{-1} = L^2 t^{-2}$]. The other quantities are the pipe diameter D [L], the Fanning friction coefficient f [dimensionless], and the heat flux density $q(x)$ [$EL^{-2}t^{-1} = Mt^{-3}$] from the pipe wall into the fluid. In what follows we shall use subscript '0' to refer to upstream conditions and subscript 'e' to refer to conditions at the exit.

It is convenient to eliminate the related fluid density $\rho=1/v$ and velocity $w=Gv$ in favour of the above variables. The total mass flow rate from the breach is GA where $A=\pi D^2/4$ but again it is convenient to work almost exclusively in terms of quantities per unit cross-sectional area of pipe.

These equations together with the equations of state (3.1), (3.4) form a closed set for the five quantities p, v, G, T, h , as long as we know the heat transfer rate q . We shall seek separate approximations within the context of regimes ii and iii above to render them more analytically tractable.

The mass integral

Before going on to look separately at the details of regimes ii and iii, let us note some of the common points of the methods which will be used. We can integrate the mass equation (4.1) along the whole length of the pipe to obtain the rate of change of the total mass of fluid (per unit area) in the pipe:

$$\frac{dM}{dt} = G_0 - G_e \quad (4.4)$$

where G_0 is the in flow at the upstream end and G_e is the outflow at the breach. $G_0(t)$ will be prescribed as zero for a closed pipe or to be some rate determined by pumping. G_e remains to be predicted but will decrease monotonically in time through regimes ii and iii, and it can therefore be used as a measure of time. Similarly the pressure decreases monotonically along the pipe and at any given time can be used for much of the calculation as a measure of distance.

Our method will use the momentum and energy equations to predict the upstream and downstream pressures $p_0(G_e^2)$, $p_e(G_e^2)$ as functions of this release rate, the specific volume of the fluid along the pipe as $v(p, G_e^2)$, and ultimately the mass $M(G_e^2)$. Pressure will be related to distance by an equation involving dp/dx , and

mass to time by (4.4). The last step of the method will compute the time at which a particular value of G_e is achieved by integrating (4.4) in the form

$$t = \int_M^{M_{init}} \frac{dM}{(G_e - G_0)} \quad (4.5)$$

where subscript "init" refers to the initial value at time $t=0$. Some care is required with the singularity when G_0 is non-zero, but at least the singularity is explicit and appropriate mathematical techniques can be adopted to handle it. This integral can be performed through regimes ii and iii after all separate considerations have been taken into account in obtaining $M(G_e^2)$.

5 Flow in Regime ii - Modelling the Flash Front

The flow model in regime ii

The essential features of this regime ii are the initially choked exit pressure and the flash front moving into the saturated liquid zone.

We expect the flash front to be moving slowly compared with the fluid velocity out of the exit and it is therefore appropriate to consider the flow in the two-phase zone in this regime as quasi-steady. Considering for the moment the case $q=0$ of zero heat transfer from outside the fluid, the mass, momentum and energy equations in the steady flow limit (obtained by setting the $\partial/\partial t$ terms above to zero) tell us that G is uniform along the flow in the two-phase zone, that

$$\frac{d(G^2 v + p)}{dx} = -2f \frac{G |G| v}{D} \quad (5.1)$$

and that

$$h + G^2 v^2 / 2 = E \quad (5.2a)$$

is also uniform along the flow. These equations are used to describe Zone 2 in Figure 1 while the saturated liquid in Zone 1 remains at rest uniformly and constantly at (p_0, T_0) . Here E is a constant of integration - the "stagnation enthalpy" - ie the enthalpy at a (usually fictitious) point where the velocity $w=Gv$ is zero.

Heat transfer from the pipe wall

We have explored various ways of allowing for heat transfer from the pipe wall. Our conclusions are that heat from the steel pipe itself is relatively readily available to aid vaporisation, but that heat from outside the pipe is much less readily available. We can idealise this situation in the case of quasi-steady flow by demanding that the local temperature in the steel wall closely matches the neighbouring fluid temperature at all times. Thus we give the fluid immediate access to any heat stored within the pipe wall.

Implementing this approximation in the model results in a modification to the energy equation to give

$$h' + G^2 v^2 / 2 = E \quad (5.2b)$$

where h' is an effective specific enthalpy given by

$$h'(v,T) = h(v,T) + \frac{\rho_s}{\rho_L} \frac{4Y}{D} c_s T \quad (5.3)$$

in which ρ_s , c_s , Y are the density, specific heat, and thickness of the steel pipe wall. In what follows we'll drop the primes and write simply "h" with the understanding that if heat transfer is to be included in regime ii, then the last term of (5.3) must be included in the definition.

Boundary conditions - choked and unchoked flow

The upstream boundary conditions, applied to Zone 2 in Figure 1, give values T_0 , $p=p_{\text{sat}}(T_0)$, $v_L(T_0)$, $h_L(T_0)$ at the flash front (b) in figure 1. The as yet unknown mass flux density G is supplied by a downstream boundary condition:

$$p_e = \max(p_a, p_{\text{choke}}(G)) \quad (5.4)$$

which we shall now explore.

Choked flow in the simple, steady, one-dimensional flow model presented above is most clearly understood by writing the steady momentum equation in the form:

$$\left[G^2 \frac{dv}{dp} + 1 \right] \frac{dp}{dx} = -2f \frac{G |G| v}{D} \quad (5.5)$$

For any given G , sooner or later (in general) there will occur a point in the flow where dv/dp reaches a value of $-1/G^2$. At this point the fluid has reached its local speed of sound and dp/dx will be negative and infinite (although p will be finite). We cannot advance beyond this point in x and if this happens in the pipe, it can only be at the exit. Thus the choke equation

$$\frac{dv}{dp} = -\frac{1}{G^2} \quad (5.6)$$

yields a choke pressure $p_{\text{choke}}(G)$ which gives the downstream boundary condition (5.4). In order to evaluate the choke pressure we first need the function $v(p)$.

The steady solution

The energy equation (5.2) and the equation of state for enthalpy (3.4) may be combined to eliminate h yielding:

$$v = \frac{1}{G^2} \left[-\phi + \sqrt{\phi^2 + 2G^2(E + v_L\phi - h_L)} \right] \quad (5.7)$$

Knowledge of how the thermodynamic functions ϕ , v_L , and h_L depend on T , and hence through the saturation curve on p , makes this a prediction of the variation $v(p)$ along the pipe. It is well defined at small mass flow rates with the limit

$$v \xrightarrow{G \rightarrow 0} \frac{E + v_L\phi - h_L}{\phi} \quad (5.8)$$

The assumption or derivation of a profile $v(p)$ along the pipe is crucial to all analytic solutions of pipe flow problems and we shall discuss the relationship of this one with those of Fannelop and Ryming (1982) and of Leung and collaborators - eg Leung (1986) - below.

With the profile $v(p)$ we can use the momentum equation to derive the variation $p(x)$ of pressure along the pipe implicitly in the inverse form $x(p)$

$$x(p) = \frac{D}{2f} \left[\frac{1}{G^2} \int_{p_e}^p \frac{dp}{v} - \ln \left(\frac{v_e}{v} \right) \right] \quad (5.9)$$

where in this equation it has proved convenient to change to measuring distance x upstream from the exit point to the point where the pressure and volume are (p,v) . In general the integral can be done numerically very quickly. We can thus, given the mass flux density G , determine the profile of all relevant quantities along the pipe through the two-phase zone. In particular the length L_2 of the two-phase zone can be found for given G^2 , by integrating up to p_0 .

The choke pressure and the initial flow rate

As we have already seen, we also need the pressure derivative dv/dp , and with a little algebra this turns out to be

$$\frac{dv}{dp} = - \frac{1}{(G^2 v + \phi)} \left[(v - v_L) \frac{d\phi}{dp} + \frac{dh_L}{dp} - \phi \frac{dv_L}{dp} \right] \quad (5.10)$$

where of course all pressure derivatives on the right hand side may be written as temperature derivatives using

$$\frac{d}{dp} \equiv \frac{dT}{dp} \frac{d}{dT} \equiv \frac{T}{\phi} \frac{d}{dT} \quad (5.11)$$

Fluid dynamically they are just total derivatives along the flow; thermodynamically they are partial derivatives along the saturation curve, and are assumed known for whatever substance is being considered.

The choke condition (5.6) therefore relates the choke pressure implicitly to G through

$$\left[(v - v_L) \frac{d\phi}{dp} + \frac{dh_L}{dp} - \phi \frac{dv_L}{dp} - v \right] G^2 = \phi \quad (5.12)$$

This can be solved for known thermodynamic properties and using $v(p,G^2)$ as given above to give $p_{\text{choke}}(G^2)$ or $G^2(p_{\text{choke}})$. A special case is the initial flow rate which corresponds with a choke pressure equal to the initial pressure $p_0 = p_{\text{sat}}(T_0)$ and with the specific volume being that of the saturated liquid. In this case we find explicitly

$$G_{\text{init}}^2 = \frac{\phi^2}{\left(c_L T - \phi \left[T \frac{dv_L}{dT} + v_L \right] \right)} \quad (5.13)$$

where $c_L = dh_L/dT$ is the liquid specific heat (modified as above by the heat-transfer term when appropriate). This constitutes a prediction of the initial mass ejection rate (per unit pipe area) which depends only on the initial temperature and the thermodynamic properties of the fluid in the pipe.

Evolution of the flow

Our quasi-steady model for the flow evolution in regime ii is as follows. We start with pure saturated liquid at T_0 , p_0 and the initial flow rate defined above. The length of the two-phase zone, Zone 2 in figure 1, is initially zero.

The flow rate will drop in time. Consider the situation when it has dropped to an arbitrary value G . The upstream pressure, temperature, and specific volume are unchanged. The new choke pressure can be computed from (5.12) with $v(p)$ given by (5.7) and with a stagnation enthalpy

$$E = h_L(T_0) + G^2 v_L^2(T_0) / 2 \quad (5.14)$$

defined by the upstream end of Zone 2, and the exit pressure is then given by (5.4) and the flow will unchoke when G falls to a point where the choke pressure is atmospheric pressure. The volume profile with pressure (5.7) can be used in (5.9) to define the profiles in x . In particular (5.9) can be used to yield the distance to the point $(p_0, T_0, v_L(T_0))$ giving a prediction of the length L_2 of the two-phase zone when the given value of G is achieved.

Finally the total mass of fluid (per unit cross-sectional area) in the pipe is found from

$$\begin{aligned} M &= \int_0^L \frac{dx}{v} = \frac{(L - L_2)}{v_l(T_0)} + \int_{p_e}^{p_0} \frac{dx}{dp} \frac{dp}{v} \\ &= \frac{(L - L_2)}{v_l(T_0)} + \frac{D}{2f} \left[- \left(\frac{1}{v_{L0}} - \frac{1}{v_e} \right) + \frac{1}{G^2} \int_{p_e}^{p_0} \frac{dp}{v^2} \right] \end{aligned} \quad (5.15)$$

and the time taken to arrive at this value of G^2 can be evaluated from (4.5).

Let us note that this quasi steady approximation is formally justified as long as the liquid erosion velocity, dL_2/dt , which is of order $Gv_L(T_0)$, is much smaller than the fluid velocity Gv_e at the exit, that is to say as long as $v_e \gg v_L$. For the most part this is satisfied well enough.

6 Flow in Regime iii - Further Evolution

The time-dependent flow problem

In the case of zero inflow the flash front will encounter the upstream end of the pipe and initiate regime iii. In regime iii the upstream pressure and temperature will also drop and the flow is intrinsically time-dependent, and so we start again from equations (4.1, 4.2, 4.3).

Regime iii does not have the distinct liquid zone and in fact can be thought of as being in many ways very similar to the time-dependent single-phase gas pipeline problem analysed by Fanelop and Ryhming (1982) but with a rather different equation of state. It is therefore useful to follow the method of Fanelop and Ryhming as far as possible in developing our regime iii model.

The method involves estimating the approximate profiles $p(x,t)$, $G(x,t)$ along the pipe, specified as given functions of x (but with unknown time dependence at this stage) to satisfy appropriate boundary conditions, using them to integrate the above equations over x , and obtaining ordinary differential equations in time, which can then be solved for the pressure and flow rates at the ends of the pipe. Encouragingly, Fannelop and Ryhming showed that the results do not depend sensitively on the details of the chosen profiles.

Approximating the energy equation

In attempting this here, we rapidly discovered that there is one feature of single phase gas flow which gave Fannelop and Ryhming an advantage that we do not share in the two-phase analysis. It turns out that the gas flow case can be considered isothermal - the temperature is approximately constant along the pipe. Fannelop and Ryhming knew this from experience with actual gas pipelines, and in the course of this work we have now also found a formal mathematical derivation of this. Essentially heat transfer into the fluid ensures that any temperature change (from a compressor) near the end is compensated by a return to ambient temperature within a finite distance. For a sufficiently long pipeline, then, the temperature variations occur over a very small fraction of the total length and contribute minimally to the profile integrals along the whole length of the pipe. The specific volume is simply proportional then to $1/p$ from the isothermal gas equation of state, and the energy equation reduces to $T=T_0$.

In the two phase case, not only is the form of $v(p)$ rather more complicated, but the energy equation cannot reduce to anything like $T=T_0$. This last is very clear if homogeneous equilibrium holds (as indicated by fig 3) as a pressure drop along the pipe *must* be accompanied by a temperature drop as $p=p_{\text{sat}}(T)$ holds at every point along the pipe.

If we are to make further progress with an analytic model of time-dependent two-phase flow, we first need an analogously simple approximation of the energy equation, but one which can support homogeneous equilibrium flow. The key to generalising the model of Fannelop and Ryhming to two-phase flow is to observe that for gas flow, one might just as well have specified a constant specific enthalpy h . Because, for simple gases, $h \sim T$ it makes no difference whether one chooses constant h or constant T . It makes a big difference for two phase flow however. And because the reason for the constant T is largely to do with heat transfer, then constant enthalpy is in fact a more physical way of thinking of it. We can adopt this uniform enthalpy hypothesis as an estimate of the behaviour of two-phase flow down long pipelines where any heat transfer may keep the overall specific enthalpy relatively constant along the pipe. In fact we'll go one step further and write

$$h + G^2 v^2 / 2 = E \tag{6.1}$$

for some uniform E . This makes little difference compared with assuming uniform h as the velocity (Gv) term is generally small compared with h (except perhaps very close to a choke) especially by the time we're in regime iii, as by then the outlet velocity will have decreased very much from its initial value.

Adopting this equation also conveniently gives us (with the homogeneous equilibrium thermodynamics) exactly the same $v(p)$ profile (5.7) as we had before, except that we are now considering that G may vary along the flow. However, consistently with the above remarks on the velocity terms in the simplified energy equation, we expect $v(p)$ to be very close to its limiting ($G \rightarrow 0$) form throughout regime iii, and so any variation of G along the flow will make little difference to $v(p)$.

For the same reason, in practice the stagnation enthalpy E varies only very slightly in time throughout regime ii, and so, consistently both with regime ii and with the gas flow analysis we shall take it to be constant in time through regime iii of the two-phase flow, and give it whatever value it adopts at the end of regime ii.

It is also important to note that h really is just the fluid enthalpy here. The arguments used in regime ii to include heat transfer by means of a pipe term added to the enthalpy only apply to quasi-steady flow and break down for fully time dependent flow. On the other hand we are allowing for heat transfer here as it is only the existence of heat transfer which allows us to argue for constant specific enthalpy in the gas flow case, and by extension for constant E here.

Approximating the momentum equation

Fannelop and Ryhming (1982) neglect the $\partial G/\partial t$ in the momentum equation, arguing for a balance of pressure gradient and friction. With this approximation they integrate the momentum equation (4.2) along the length of the pipe. This gives

$$\int_{p_e}^{p_0} \left(1 + \frac{d(G^2 v)}{dp} \right) \frac{dp}{v} = \frac{2f}{D} \int_0^L G^2 dx \quad (6.2)$$

The profile method can be illustrated by approximating the mass flux profile as

$$G^2 = G_0^2 + [G_e^2 - G_0^2] K(x/L) \quad (6.3)$$

for some function K with $K(0)=0$, $K(1)=1$. And so the integral so the right hand side of (6.2) can be simply estimated. In fact we shall consider the case $G_0=0$ as being the only one of interest for regime iii and define k as the integral of $K(z)dz$ from 0 to 1. The constant k will be of order 1.

It is worth noting in fact that Fannelop and Ryhming also neglected the $d(G^2 v)/dp$ term on the left of the momentum equation. It tends to be small except near the choke and in the gas flow problem they were considering unchoked flow for much of the time. We can do slightly better in the case of choked flow (and a relatively large range of specific volume) if we approximate

$$\frac{d(G^2 v)}{dp} \approx \lambda G_e^2 \frac{dv}{dp} \quad (6.4)$$

with λ a constant of order 1. (Fannelop and Ryhming's approximation is recovered by setting $\lambda=0$.) With these approximations equation 6.2 reduces to

$$\frac{1}{G_e^2} \int_{p_e}^{p_0} \frac{dp}{v} - \lambda \ln \left(\frac{v_e}{v_0} \right) = 2kf \frac{L}{D} \quad (6.5)$$

In principle the free constants k , λ reflect our lack of knowledge of the profiles. However, in practice we can demand that they be such as to make the results behave as continuously as possible through the transition from regime ii to regime iii. This effectively fixes $k=1$ and $\lambda=1$ and is consistent with assuming that changes in v along the pipe have much more important effect on the momentum balance than changes in G^2 . It also means that the choke pressure may be estimated exactly as in regime ii.

The momentum equation now relates the two end pressures p_e and p_0 to the outflow G_e . We already have $v(p)$ and so we can make use of it without any further profile assumptions.

However, as before, we need to know about $v(p(x))$ to evaluate the mass release rate and here an assumption about the pressure profile in x is required. Our first approximation followed the gas flow analysis exactly: we set $p(x)$ to be quadratic in x with the appropriate values at the ends and $dp/dx=0$ at the closed upstream end. We discovered, however, that the isothermal gas equation of state $v \sim 1/p$ makes the mass integral uniquely tractable, and failed to make analytic progress to the same extent with our more

complicated two-phase $v(p)$. The maths gets complicated very rapidly here, which rather defeats the object of the approximation: choosing a "simple" profile to get results simply. Therefore, noting Fannelop and Ryhming's observation that the results are not strongly sensitive to the precise profiles chosen, it is worth asking if there is an appropriate choice which simplifies the calculation in this case.

In fact there is. Let us approximate the pressure profile $p(x)$ by one which satisfies

$$\frac{d(\lambda G_e^2 v + p)}{dx} = -2kf \frac{G_e |G_e| v}{D} \quad (6.7)$$

We don't have an explicit form for it and it only approximately satisfies $dp/dx=0$ at the closed end, but it does have the right values at each end as well as the correct analytic behaviour near the choke, and again it is very convenient in that it gives complete continuity of the mass integral (with the above values of k and λ) through the transition from regime ii to regime iii. With this profile, the mass integral is in fact

$$M = \int_0^L \frac{dx}{v} = \int_{p_e}^{p_0} \frac{dx}{dp} \frac{dp}{v} = \frac{D}{2kf} \left[-\lambda \left(\frac{1}{v_0} - \frac{1}{v_e} \right) + \frac{1}{G_e^2} \int_{p_e}^{p_0} \frac{dp}{v^2} \right] \quad (6.8)$$

and can be evaluated as before from our knowledge of the specific volume profile $v(p)$.

Evolution of the flow in regime iii

As earlier we can evolve the flow by decreasing the mass flux density G to a new value. The stagnation enthalpy E is assumed not to change. The exit pressure is computed exactly as in (5.4). Then (6.5) is solved for the new upstream pressure p_0 , using the same volume profile $v(p, G_e)$ as before. The mass of fluid in the pipe is given by the integral (6.8) and finally the time at which this new value of G is achieved is computed from (4.5) (with $G_0=0$).

This completes the main description of the model. It remains only to compare it with experimental data.

7 Thermodynamic and fluid dynamic properties

In order to evaluate solutions of the equations we need a number of thermodynamic properties of the fluid in the pipe. Our objective is to access the DNV properties data base and retrieve whatever is needed, but at the time of writing this has not been completed.

In case a substance is not present in the data base, an alternative simplified thermodynamic option is available which we shall use here for propane in order to compare with the Isle of Grain trials. We approximate the liquid specific volume v_L as constant so that $dv_L/dT=0$. The liquid specific enthalpy is modelled as $h_L=c_L T$ with constant specific heat c_L . The vapour pressure curve is fitted in the appropriate range by the Clausius-Clapeyron form $p=A \exp(-B/T)$ so that $\phi=pB/T=p \ln(A/p)$. From these we can derive all the required properties.

For propane the values $v_L=2.07 \cdot 10^{-3} \text{ m}^3/\text{kg}$, $c_L=2616 \text{ J/K/kg}$, $A=21244 \text{ bar}$, $B=2299\text{K}$ provide a reasonable thermodynamic description in the region of interest. (These values have been extracted from, and in the case of the vapour pressure curve fitted to, correlations given by Reid, Prausnitz and Poling (1987) and by Atkins (1994)).

The other required parameter is the Fanning friction coefficient. Fannelop (1994) p115 quotes a formula used by the American Gas Association:

$$\frac{1}{\sqrt{f}} = 4 \log_{10} \left(\frac{3.7 D}{z_0} \right) \quad (7.1)$$

and recommends taking a roughness length for the inner pipe wall $z_0=1.3 \cdot 10^{-5}$ m (0.0005 inches) in the absence of any better information. Following this we estimate $f=0.0029$. (It turns out that the results - especially the mass release rate - are not enormously sensitive to this value and so we shall not, at this stage, explore further refinements.)

8 Comparison with the Isle of Grain Trials

We are now in a position to compare the model with the Isle of Grain experiments. We shall examine the full bore rupture of a 6 inch pipe - trial P42. Our comparisons with the pressure and temperature at each end of the pipe, and with the evolution of the mass inventory are shown in figures 4, 5 and 6. We emphasise that we have adjusted no free parameters to achieve the fit: thermodynamic properties of propane have been taken from books, as was the Fanning friction coefficient. Other possible parameters k and λ were fixed to achieve continuity from regime ii to regime iii as described above.

The pressure and temperature data at the closed end of the pipe are very suggestive of our regimes ii and iii, as they remain approximately constant and then start to drop at a time which matches our prediction fairly closely - especially in the pressure curve.

We have shown predictions including our heat transfer model in regime ii (bold lines) and with it switched off (thinner lines). Including the heat transfer improves the fit.

We note from the data that the open end pressure drops to around 0.5bar at the end which we regard as indicating a 0.5bar or so error on the pressure data, in view of which we believe our predictions to be very reasonable.

Temperature changes in the data take slightly longer than our model predicts and we can speculate that this may be attributable to the way in which temperature was measured. We emphasise however that the fits to pressure and temperature data are constrained in our model by the homogeneous equilibrium approximation, which we believe to be strongly justified by the overall data fig 3.

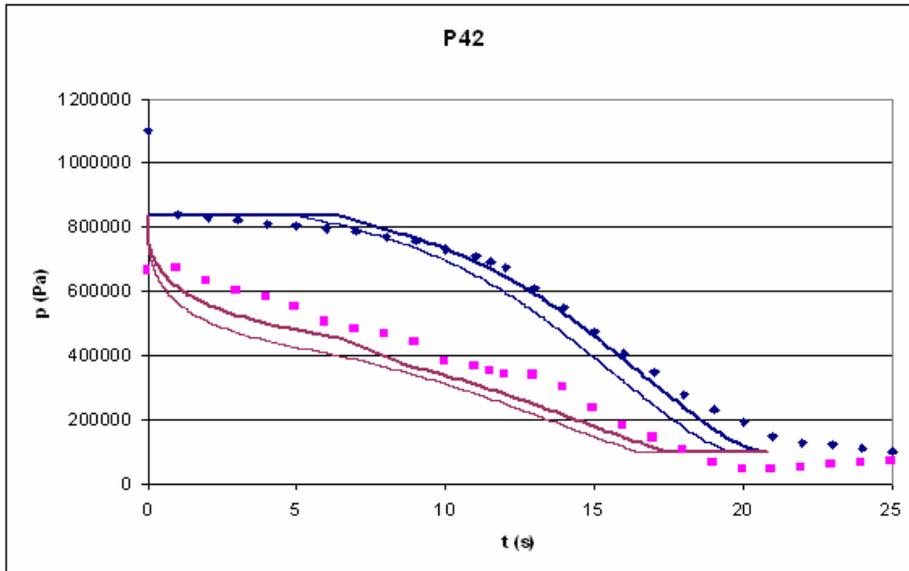


Fig. 4 The measured pressure at the open (squares) and closed (diamonds) ends of the pipe in Isle of Grain trial P42. The lines are the model predictions with heat transfer (thick lines) and without (thin).

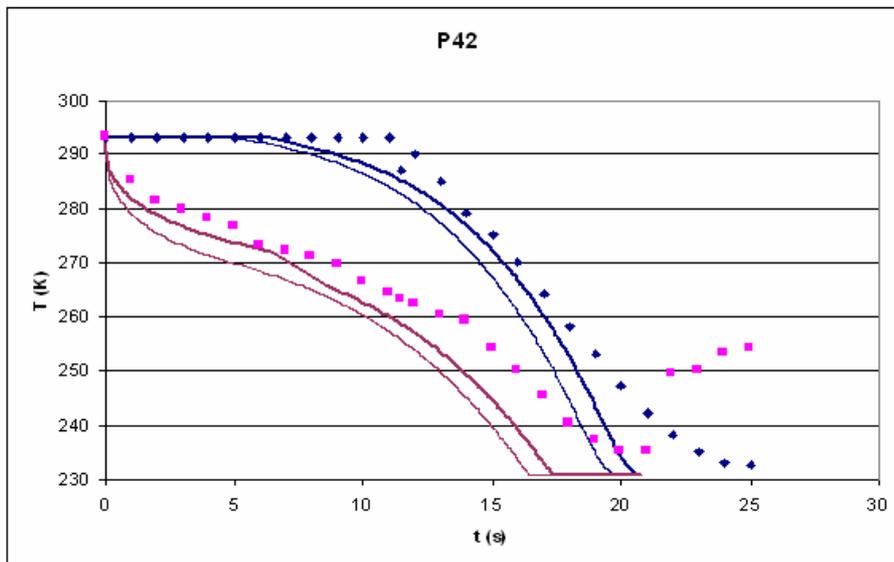


Fig. 5 The measured temperature at the open (squares) and closed (diamonds) ends of the pipe in Isle of Grain trial P42. The lines are the model predictions with heat transfer (thick lines) and without (thin).

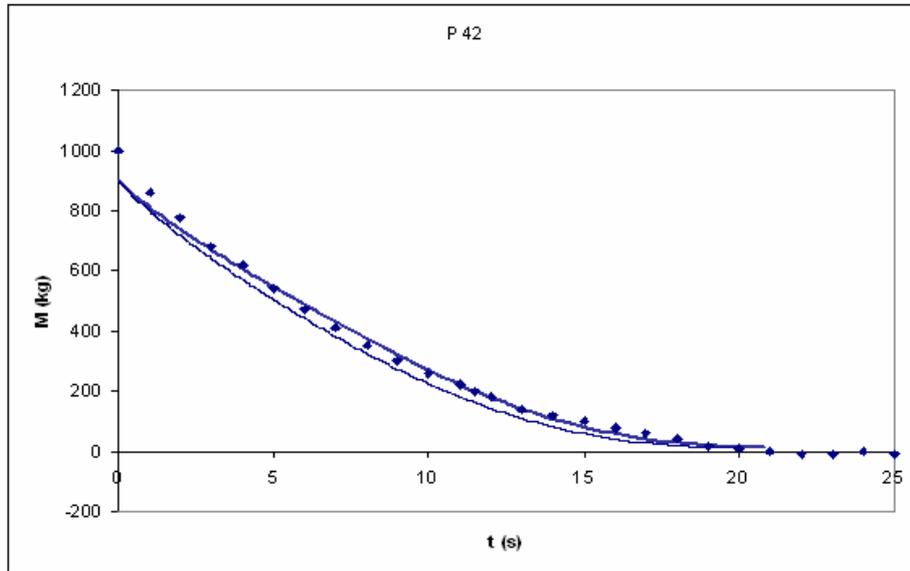


Fig. 6 The measured mass content of the pipe (points) and the model predictions with heat transfer (thick lines) and without (thin) in Isle of Grain trial P42.

One of the quantities of prime interest, the time taken to empty the pipe completely, is predicted very accurately and appears to be quite insensitive either to detailed assumptions about heat transfer, or to the precise value of the Fanning friction coefficient.

Although we regard these predictions as eminently acceptable, especially considering that we adjusted no free parameters, it is interesting to think how one might make them even better. In fact this looks difficult. It would be nice if our initial mass release rate were slightly steeper. However this is predicted absolutely from the thermodynamic properties of propane and does not depend on anything else (and in particular it does not depend on the Fanning friction coefficient at all). Allowing for a small positive dv_L/dT might increase it slightly and we'll find out when we couple in the more sophisticated thermodynamic model, but we don't expect a big effect. Otherwise, looking at regime ii where our prediction of the upstream pressure will not change, it seems we need a slightly higher downstream pressure and a slightly higher mass release rate to make any improvement in the fit. We have already argued that adjusting the friction coefficient will not achieve this (in the early stages) and therefore it is difficult to see how we can get mass out of the pipe faster with an overall smaller pressure gradient. In fact the predictions appear remarkably stable to any small adjustment we might have the freedom to make. This is already evident in the relatively small difference made when one allows for, or neglects, heat transfer. A slightly higher value of the Fanning friction coefficient may slightly improve the pressure and temperature curves at later times, but may also make the overall mass curve slightly worse, and so we are content, for the moment, to leave it at the prescribed value.

9 Discussion and Conclusions

We have presented a one-dimensional model of two-phase fluid flow down a pipe following an accidental breach. It is applicable to pipelines containing fluids with a normal boiling point below ambient temperature.

We have used a quasi-steady flow model for the movement of flash front into the fluid, followed by a generalisation of the method developed by Fannelop and Ryhming (1982) for long gas pipelines. The resultant model predicts the evolution of the system including the mass flux out of the breach with essentially no unknown free parameters. Comparisons done thus far with the Shell Isle of Grain experiments are extremely encouraging.

The model has points in common with both the gas pipeline model of Fannelop and Ryhming and with the model expounded in various papers by Leung and co-workers - for example Leung (1986). An interesting point of comparison is the variation of specific volume with pressure along the pipe. In those two models and the current one, the solution method is essentially to derive $v(p)$ from a combination of the energy equation and the equation of state (or guess it) and then use it in the momentum equation to relate the upstream and downstream pressures to the release rate. As noted above, Fannelop and Ryhming have an isothermal gas approximation so that $v=b/p$ for some constant b . Leung's two-phase model uses the form $v=a+b/p$ with constants a and b , which are chosen to optimise this form at and near the high pressure end of the pipe. The current model derives $v(p)$ from the energy equation and the homogeneous equilibrium model of state, and the result in fact also depends on the mass flux $v(p,G^2)$ although the G^2 dependence may not always be strong and will diminish with time. But even at small G^2 the specific volume turns out to have a more complicated dependence on p than can be accommodated by Leung's approximation, and we therefore believe our model to be much more appropriate when the pressure difference between the ends of the pipe is more than a factor of two or so. Also our model, via its function $v(p,G^2)$ actually predicts the choke pressure, something which must be done separately if a simpler approximate form is used.

Richardson and Saville (1996) have an apparently much more complex model which accounts for mixtures of hydrocarbons, something which they imply is important for understanding the flow in the Isle of Grain experiments. In contrast we find that a very much simplified model (both dynamically and thermodynamically) is capable of yielding excellent results. In fact if we examine Richardson and Saville's fit to the Isle of Grain trial discussed above, we see no better a fit to the data than is given by our own model. This may provide added support for our conclusion that it is difficult to improve the fit to the mass and pressure curves simultaneously, but in any event we conclude that the accuracy of our model is very comparable with that of the more complex model of Richardson and Saville.

Our model is still under development and we aim to compare with a larger number of experiments (as already done by Richardson and Saville with their model) including ones with an aperture smaller than the pipe area. In this last case the flow rate G_x through the aperture of area A_x will be related to the flow rate G immediately behind the aperture by conservation of mass flux, $G_x A_x = GA$, and the only essential change to the model is that the outlet pressure is modelled as

$$p_e = \max(p_a, p_{choke}(GA/A_x)) \quad (9.1)$$

in place of (5.4). We look forward to testing it for this case.

Finally, coming back to one of our prime motivations, this model can be used to predict the behaviour of ruptured ethylene pipelines. The complicating factors of ethylene are that much higher pressures are involved and that the critical temperature is at 10 Celsius - right in the normal range of ambient temperatures. Possibly large pressure drops are not a problem for our model and re-examining our formulation we see that none of the thermodynamic quantities needed are singular at the critical point. Thus the whole model will go through as long as we can expect the temperature at the start of regime ii to be below the critical temperature. The simplified thermodynamic correlations (constant liquid specific heat and specific volume) are not appropriate in the vicinity of the critical point, but this will not prove a problem when the model can interrogate a thermodynamic database for these quantities.

The model is quick to use. If one demands that the time range is split into 100 intervals, the model takes less than 1 second to run on a 300MHz Pentium II computer, and is therefore ideally suited to hazard analysis for a wide range of two-phase pipeline problems. It is planned to include this model in a future version of the DNV consequence modelling package PHAST and in the risk analysis package SAFETI.

Nomenclature

A [L²] inner pipe cross-sectional area

A	$[ML^{-1}t^{-2}]$	coefficient in the Clausius-Clapeyron vapour pressure correlation
B	[T]	coefficient in the Clausius-Clapeyron vapour pressure correlation
c	$[EM^{-1}T^{-1}]$	specific heat
D	[L]	inner pipe diameter.
E	$[EM^{-1}=L^2t^{-2}]$	(specific) stagnation enthalpy
f		Fanning friction coefficient
G	$[ML^{-2}t^{-1}]$	mass flux density
h	$[EM^{-1}=L^2t^{-2}]$	specific enthalpy
k		a profile integral (=1)
L	[L]	pipe length
M	$[ML^{-2}]$	mass of fluid in the pipe per unit cross-sectional area
p	$[ML^{-1}t^{-2}]$	pressure
q	$[EL^{-2}t^{-1}=Mt^{-3}]$	heat flux density from the pipe wall
t	[t]	time
T	[T]	temperature
v	$[L^3M^{-1}]$	specific volume
w	$[Lt^{-1}]$	fluid flow velocity
x	[L]	distance (downstream from the upstream end of the pipe or upstream from the downstream end of the pipe according to convenience).
Y	[L]	pipe wall thickness
z_0	[L]	inner pipe wall roughness length
η		liquid mass fraction (quality)
λ		a profile parameter (=1)
ρ	ML^{-3}	density
ϕ	$[ML^{-1}t^{-2}]$	T dp/dT along the vapour pressure curve

Subscripts:

0	Upstream end of the flow
2	pertaining to the two-phase flow zone
a	ambient/atmospheric
e	Downstream (exit) end of the flow
init	Initial conditions (at time t=0)
L	liquid phase
v	vapour phase
s	steel pipe
sat	at saturation (on/along the vapour pressure curve)

The dimensions given in the above table are L=length, t=time, M=mass, T=temperature, and for convenience $E \equiv ML^2t^{-2}$ =energy. All of the equations in this paper are dimensionally correct and so quoting actual units here would be superfluous. (Any units, SI, British Imperial, or American, may be used to evaluate the formulae as long as they are applied consistently. Where values are quoted we have done so in SI.)

Acknowledgement

DMW and TKF gratefully acknowledge the support of Det Norske Veritas Ltd, who commissioned this model development from us. All of the authors gratefully acknowledge the support of RIVM (the Dutch National Institute of Public Health and the Environment) who set the original agenda for this work.

References

P W Atkins, (1994) "Physical Chemistry" 5th Edn OUP

L T Cowley and V H Y Tam (1988) "Consequences of pressurised LPG releases: the Isle of Grain full scale experiments", GASTECH 88, 13th International LNG/LPG Conference, Kuala Lumpur

T K Fannelop (1994) "Fluid Mechanics for Industrial Safety and Environmental Protection", Elsevier.

T K Fanneløp and I L Ryhming (1982) "Massive release of gas from long pipelines" AIAA Journal of Energy **6** pp132-140

J C Leung (1986) "A generalised correlation for one-component homogeneous equilibrium flashing choked flow" AIChE Journal **32** pp1743-1746

R C Reid, J M Prausnitz, and B C Poling (1987) "The Properties of Liquids and Gases" 4th Edn McGraw Hill.

S M Richardson and G Saville (1996) "Blow-down of LPG Pipelines". Trans. I.Chem.E **74B** pp235-244